

**Abstract.** An alternative method of assigning a homotopy type to a Grothendieck topos is described. This method yields trivial results in almost all cases which have been previously studied, e. g. for toposes of sheaves on any Hausdorff topological space. On the other hand, it is shown that in many cases when the standard method gives trivial results – e. g. for some typical “gros” toposes (in the sense of Grothendieck and Lawvere), the alternative method yields new homotopy-theoretic information.

The obtained homotopy types are related to some previous investigations. It is shown that  $\Gamma$ -rings recently assigned to simplicial algebraic theories by S. Schwede (*Stable homotopy of algebraic theories*, Topology 40 (2001), no. 1, 1–41) can be viewed as stabilizations for the homotopy types of their classifying toposes. This implies that (co)homology of algebraic theories previously studied by T. Pirashvili and the author describes cohomology of these “gros” homotopy types. It turns out that these cohomologies can be defined for general toposes and yield some classification possibilities, at least in lower dimensions.

Some examples are given hinting at the possibility of modelling homotopy types of some ring spectra related to  $K$ -theory and elliptic cohomology by “gros” homotopy types.